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Large Numbers and the Time Variation of Physical Constants

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We consider a cosmological model consistent with observation which not only explains the well-known large-number coincidences, but also deduces the values of the mass, radius, and age of the universe, the Hubble constant and the cosmological constant, a relation between the pion mass and the Hubble constant known so far only as a mysterious empirical coincidence, and other features. This model predicts an ever-expanding universe, as indeed latest astrophysical data indicate.

1. INTRODUCTION

Dirac's Large Number Hypothesis (LNH) has been much written about (Dirac, 1938; Barrow and Tipler, 1986; Weinberg, 1972; Rees *et al.*, 1974; Berman, 1992, 1996; Berman and Gomide, 1994; Beesham, 1994a,b). This is based on apparently mysterious ratios of certain physical constants which coincide or show a relationship. Let us start with

$$N_1 = \frac{e^2}{Gm^2} \approx 10^{40} \tag{1}$$

where m is the pion mass, the pion being a typical elementary particle, this being the ratio of the electromagnetic and gravitational forces, and

$$N_2 = \frac{cT}{l} \approx 10^{40} \tag{2}$$

where T is the age of the universe and l the pion Compton wavelength.

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In this light, the LNH can be stated as follows (Beesham, 1994a): "Any two of the very large dimensionless numbers occuring in nature are connected by a simple mathematical relation, in which the coefficients are of the order of magnitude unity."

An application of this to (1) and (2) means that their equality is not accidental, but rather leads immediately to Dirac's well-known relation,

$$G \propto T^{-1} \tag{3}$$

Dirac's approach further leads to

$$R \propto T^{1/3} \tag{4}$$

which appears to be inconsistent (Weinberg, 1972; Ma, 1995).

Another "accidental" relation is

$$m \approx \left(\frac{\hbar^2 H}{Gc}\right)^{1/3} \tag{5}$$

As observed by Weinberg (1972), this is in a different category and is unexplained: it relates a single cosmological parameter H to constants from microphysics.

In the spirit of LNH, one could also deduce that (Berman, 1996)

$$\rho \propto T^{-1} \tag{6}$$

and

$$\Lambda \propto T^{-2} \tag{7}$$

where ρ is the average density of the universe and Λ is the cosmological constant.

It may be mentioned that attempts to generalize or modify the LNH have been made (e.g., Ma, 1995; Carvalho, 1995), but without gaining much further insight.

2. FLUCTUATIONS

We now deduce (3) and (5)–(7) from an alternative standpoint. Moreover in place of the troublesome relation (4), we will get a consistent equation. Our starting point is the zero-point field (ZPF). According to QFT, this field is secondary, while according to stochastic electrodynamics (SED), this field is primary.

We observe that the ZPF leads to divergences in QFT (Feynman and Hibbs, 1965) if no large-frequency cutoff is arbitrarily prescribed, e.g., the Compton wavelength. On the contrary, we argue that it is these fluctuations within the Compton wavelength and in time intervals $\tau \sim \hbar/mc^2$ which create the particles. Thus, choosing the pion again as a typical particle, we get (Feynman and Hibbs, 1965; Sidharth, 1997a)

$$(\text{energy density of ZPF}) X l^3 = mc^2 \tag{8}$$

Further, as there are $N \sim 10^{80}$ such particles in the universe, we get,

$$Nm = M \tag{9}$$

where M is the mass of the universe.

In the following we will use N as the sole cosmological parameter.

Equating the gravitational potential energy of the pion in a three-dimensional isotropic sphere of pions of radius R, the radius of the universe, with the rest energy of the pion, we can deduce the well-known relation

$$R = \frac{GM}{c^2} \tag{10}$$

where M can be obtained from (9).

We now use the fact that the fluctuation in the particle number is of the order $\sqrt{N}$ (Hayakawa, 1965; Huang, 1975; Sidharth, 1997b), while a typical time interval for the fluctuations is $\sim \hbar/mc^2$ as seen above (that is, particles induce more particles by fluctuations). This leads to the relation via $dN/dt = \sqrt{N}/\tau$ (Sidharth, 1997a)

$$T = \frac{\hbar}{mc^2}\sqrt{N} \tag{11}$$

where T is the age of the universe, and, using (10),

$$\frac{dR}{dt} \approx HR \tag{12}$$

while from (12), we get the cosmological constant as

$$\Lambda \approx H^2 \tag{13}$$

where H in (12) can be identified with the Hubble constant, and from the above can be seen to be given by

$$H = \frac{Gm^3 c}{\hbar^2} \tag{14}$$

Equations (10) and (11) show that in this formulation, the correct radius and age of the universe can be deduced, given N as the sole cosmological or large-scale parameter. Equation (13) for Λ is consistent and exactly agrees with an upper limit deduced for it (Misner *et al.*, 1973). Equation (14) is identical to equation (5).

In other words, equation (5) is no longer a mysterious coincidence, but rather a consequence.

To proceed, we observe that the fluctuation of $\sim\sqrt{N}$ (due to the ZPF) leads to the empirically well known and apparently mysterious relation (1) (Sidharth, 1997a; Hayakawa, 1965) with $N_1 = \sqrt{N}$, whence we get

$$R = \sqrt{N}l \tag{15}$$

If we combine (15) and (10), we get

$$\frac{Gm}{lc^2} = \frac{1}{\sqrt{N}} \tag{16}$$

If we combine (16) and (11), we get Dirac's original equation (3). It must be mentioned that, as argued by Dirac (cf. Melnikov, 1994) we treat G as the variable, rather than the quantities m, l, c, and $\hbar$ (which we will call microphysical constants) because of their central role in atomic (and subatomic) physics.

Further, using (16) in (1), with $N_1 = \sqrt{N}$, as pointed out before (15), we can see that the charge e also is independant of time or N. So e also must be added to the list of microphysical constants.

Next, if we use G from (16) in (14), we can see that

$$H = \frac{c}{l}\frac{1}{\sqrt{N}} \tag{17}$$

Thus, apart from the fact that H has the same inverse time dependance on T as G, (17) shows that, given the microphysical constants and N, we can deduce the Hubble constant also, as from (14).

Use of (13) in (17) now gives equation (7).

Using (9) and (15), we can now deduce that

$$\rho \approx \frac{m}{l^3}\frac{1}{\sqrt{N}} \tag{18}$$

Equation (18) gives equation (6).

Next (15) and (11) give

$$R = cT \tag{19}$$

Equation (19), which is correct, differs from the troublesome Dirac dependence (4).

Finally, we observe that using M, G, and H from the above, we get

$$M = \frac{c^3}{GH} \tag{20}$$

a relation which is not only correct, but is required in the Friedman model of the expanding universe [and the steady state model also (Beesham, 1994b; Ma, 1995)]. This is entirely consistent, as we are dealing with an isotropic matter-dominated universe.

We finally make four comments:

First, in our model of particle production through fluctuations of the ZPF, equation (11) actually provides an arrow of time, at least at the cosmological scale, in terms of the particle number N.

Second, in the spirit of the uniform cosmic dust approximation, the newly created particles are uniformly spread out. In practice, as the number of the fluctuationally created particles is proportional to the square root of the particles already present, more of the new particles are created, for example near galactic centers, than in empty voids, reminiscent of the jets which are observed.

Third, the reason why the Compton wavelength emerges as a fundamental length has been seen in previous communications (Sidharth, 1997a–c).

Finally, in this model, while the mass of the universe increases as N or T^2, the volume increases as T^3, so that the mean density decreases as T^{-1} [equation (18)], unlike in the steady-state cosmology.

3. CONCLUSION

Dirac's LNH equates ad hoc, large matching numbers. The actual rationale for this is unexplained and mysterious. In the present model, particles are created by fluctuations of the ZPF. Not only does this model give, in terms of a single cosmological parameter N and microphysical constants e, m, l, c, and $\hbar$, the observed values of the mass, radius, and age of the universe, but also the values of the Hubble constant and cosmological constant and also the equality of the apparently matching large numbers, the mysterious equation (14) and the relation (20). Furthermore, the time variation of the cosmological parameters including the total particle number [equation (11)] is also a consequence of this model.

It may be pointed out that the above model answers two vexing cosmological problems. The first is the fact that some recent ground-based and Hubble Space Telescopic studies indicate that the age of the big bang universe is less than the age of certain stars (Pierece *et al.*, 1994; Freedman *et al.*, 1994). The second is the fact that recent studies indicate (Branch, 1998; Perlmutter, *et al.,* 1998) that the density of the universe is less than the critical density, so that the universe would continue to expand forever and consequently one would require the cosmological constant, both of which are in agreement with the above model.

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